Writing, an Effective Communication Tool in a Mathematics Classroom

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Mathematics is known for its numbers, symbols, shapes, and formulas. Most adults today remember their teacher lecturing at the board and then they were given multiple problems to do for homework. The students were expected to memorize the process of a concept and "spit it back out" on their homework. This strategy never gave the student an opportunity to question, explore, or communicate about a topic in mathematics. Things have change; they have changed so much that the National Council of Teachers of Mathematics (NCTM) and Common Core have adopted communication as part of the mathematical standards. Communication is an essential part of students' daily life, in and out of the classroom, why not have it in the math classroom. "When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing" (NCTM, 2000, p. 60). In Principles and Standards for School Mathematics (2000), the NCTM is asking students to write as part of their mathematics experience. It is also encouraging teachers to implement various writing opportunities into their instruction to help students organize and consolidate their mathematical thinking.

As a middle school mathematics teacher, I know students have a difficulty communicating their thoughts on anything, especially math. However, I also know that writing down thoughts is a way to analyze and reflect thus problem solve. By breaking down a problem and justifying their answer, I want to know how written communication impacts the learning of my below-average students and how they feel about their understanding of mathematics when using written communication in the classroom. I want to see how improving students' communication skills enhance their problem-solving skills. This will help me understand my students' strengths and weaknesses.

Literature Review

My review focused on how written communication affects below-average students to become better problem solvers. I found many studies and research, through ProQuest, and mathematics articles I have at home, using words such as "written communication", "mathematical reasoning", "low-level students". I noticed four major themes. First, I describe the importance of written communication. Second, account for the importance of mathematical reasoning. Third, I focus on the connection of communication and problem-solving. Fourth, I convey the connection between improvement and writing in a mathematics classroom. Finally, I conclude the review by summarizing the value of written communication in a mathematics classroom.

The Importance of Written Communication

Written communication is not only a way for students to present a solution and understanding of a mathematical concept, but it is also a way for students to express their thoughts, feelings, fears, and successes. Chelsea M. Fortescue (1994) preformed an experiment with a variety of strategies to improve students' mathematics involving writing. Fortescue (1994) states, "Using writing as a learning tool can increase comprehension in specific content areas. It not only reinforces learning of specific content but also provides rich experiences with literacy to support growth in thinking and writing" (p. 576). NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989) stresses that "communication plays an important role in helping children construct links between their informal, intuitive notions and the abstract language and symbolism of mathematics…" (pg. 26) Capacity Building Series (2010) printed an article *Communication in the Mathematics Classroom*, focusing on three communication techniques using interactive discussions, an instructional strategy, and board writing to develop students'

mathematical communication. "Through listening, talking and writing about mathematics, students are prompted to organize, re-organize and consolidate their mathematical thinking and understanding, as well as analyze, evaluate and build on the mathematical thinking and strategies of others" (p. 2). Through most writing strategies, it became evident to students that mathematical communication is not about "answering the questions using words, numbers, pictures, and symbols," instead they realize that certain writing strategies create a precise mathematical argument (p. 2). Written communication is a way for students to organize and record mathematical thinking. Because written record "enables simultaneous comparison of methods, there is potential for students to construct new mathematical ideas and deepen their mathematical understanding" (p.5). Using writing strategies will only strengthen my students' thought process, in mathematics or in any content area.

The Importance of Mathematical Reasoning

"Reasoning is the ability to think, understand, and form opinions or judgments that are based on facts" (Kaur, n.d.). Students are so quick to answer; they do this without ever asking why or analyze what they are doing it. Fortescue (1994) remembers as a student that she "…was rarely asked why a particular answer was the correct one or when I could use a problem in other areas of school or life" (p. 579). When students are asking to logically explain why and how they reach an answer, teachers can then determine the students' comprehension of the mathematical concept. Resnick (1987) states that students who are successful at mathematical reasoning are more likely to engage in reflective activity. Students who think about what they are doing and why they are doing it are more successful than those who just follow rules they have been taught. Berinderjeet Kaur (n.d.) studied strategies on communication and reasoning. He determined "no single strategy works for all students, nor even for the same student is all circumstances" (p.

104). It is because of this, that reasoning skills must be taught to improve the strategies of approaching a problem and justifying the way each individual student solves a particular problem. Once a student rationalizes their work, this can then be new knowledge for another student. According to Gunningham (2003), when students hear their classmates reasoning or thinking and solutions, they get an opportunity to value thinking as well as answers, realize that there are many paths to findings a solution, and build more complex connections between different mathematical concepts. Also when students reason their steps, Kaur (n.d.) found that "teachers support students in their efforts to explain and justify the answers by resisting the temptation to immediately highlight flaws in reasoning, praise creative approaches to problem-solving, and summarize the strategies presented and discussing the merits of using some strategies compared to the others…"(p. 109). These findings suggest that mathematical reasoning is an important part not only in mathematics, but in the communication of mathematics

Connection of Communication and Problem Solving

The connection between communication and problem-solving achievement is another theme found in the literature. Albert (2000) emphasizes that "oral language is the tool used to shape the discourse in collaborative problem solving; however, at an independent level of learning and development, writing is the tool students can use to shape their thinking" (p. 116). Like many other research reports Albert (2000) also states "the act of writing serves as a mode for students to reflect on their thinking. This method of communication allows them to convey ideas, feelings, and experiences that can lead to the development of higher cognitive functions, including critical thinking, sound reasoning, and problem-solving" (p. 109). Using writing to solve a mathematical problem can range from listing steps in the solution process to justifying why an answer is correct. NCTM (2000) suggests that writing in mathematics can also help

students "consolidate their thinking because it requires them to reflect on their work and clarify their thought about the ideas developed in the lesson" (p. 61). Therefore, student reasoning in oral or written form provides clarity to where the student's level of knowledge is and serves as a guide for future instruction.

Student Improvement with Writing in Math Class

Greenfield and Bruner (1969) observed that cultures with technologies such as written language and mathematical formalisms will "push cognitive growth better, earlier, and longer than others" (p.654). Writing allows students to evaluate their thought process, which in turn, strengthens their mathematic abilities and gives them confidence in their solutions. Evans (1984) examined the use of writing to problem solve in short response format. The findings suggest that the students with the lowest pre-test scores in the experimental group made the most gains. It was further noted from the findings that writing gave the researchers one more tool to help less capable students grow. Also, Brown (1993) explored the use of writing in his seventh-grade mathematics class to motivate below-level students. After several weeks of providing opportunities for students to creatively write addition problems and exchange them with their peers, findings revealed that students understood the concept better and experienced a feeling of achievement and success in mathematics.

Summary

My findings regarding the effects of writing in a mathematics classroom provided useful information as well as challenged which methods of writing to present to my students. The act of writing has many benefits as Miller (1991) states, "students who will not ask questions in class may express their confusion privately in writing" (p. X). Writing creates unique opportunities, such as learning mathematical content, opening a window into students' thinking, providing

teachers with information to adjust their planning, having students problem-solve while focusing on their metacognition, and facilitating conversations through a writing task.

Context

This research project will be conducted in an Algebra class at Wy'east Middle School in Vancouver, Washington. Wy'east Middle School is an underfunded suburb school accommodating approximately 1000 students in grades 6th through 8th, ages 11-14 years old. For this study, I will use my 4th period Algebra class that usually has between 28-30 students. I have decided to wait to start collecting data until November 2013 which will be about the time students will be on Unit 3 Coordinate and Functions. During the months of September and October I will be able to establish a trusting and safe environment in my classroom and learn individually students' needs. Classroom norms will be set and positive relationships between teacher/student and student/student will be formed. Also, by starting in November in Unit 3, two units would have been presented and comprehension of many mathematical standards would be assessed. I will collect data throughout Unit 3, estimating a time of 6 weeks from when I start the collection to the time I finish analyzing and reflecting on my findings. I have chosen my 4th period Algebra class because many students find it difficult to focus right after lunch. In the past, it is my 4th period class that has had the most students who have fallen behind, dropping their grade below-average, below a C. Relying upon this historical data, I am estimating that I will be asking 8-12 students to be my research subjects.

Even though I teach high school level Algebra to middle school students, I still have students within these classes that struggle with concepts in mathematics. Students who are in my Algebra classes got there one of two ways, by the advanced program system or by their state test scores. Students who are in the advance program have been in this program since 6th grade.

These students do not only take advanced Math but also advanced History and Language Arts. These 7th grade and 8th grade students also took the 8th grade math curriculum last year. On the other hand, the students who are in my Algebra class because they received a particular score on the state test, took the 7th grade math curriculum last year. These 8th grade students are skipping the 8th grade curriculum. This "jump" makes the Algebra course a little bit more difficult for these students compared to the students in the advanced program who did not "jump" an entire curriculum. If I just look at my Algebra classes, there is still a variety of mathematical abilities thus different levels of skill even though this is an advanced class. Through students' overall grades I can determine these levels and discover which students are below-average, below a C grade. Even though I believe that writing with affect all students' problem solving skills, I want to focus on how this type of communication will impact the below-average students.

Once these students are selected, they will be given a letter (Appendix A) to take home to their parents/guardians. The letter will explain the research's purpose and what actions are entailed. (Something about getting permission and assuring no harm and risks to student)

Method

I have decided to gather data in a variety of methods all through Unit 3 Coordinates and Functions to observe written improvement as well as development in problem-solving skills. Focusing on a single section of a unit will not present enough data to determine improvement. Instead, all seven sections of the unit will be taught while assessments and surveys will be analyzed.

I will start by finding out students' opinions of mathematics, writing in mathematics, and problem-solving skills in a pre-research survey (Appendix B). This will allow me to assess what they perceive their skill level to be and gauge their attitudes towards mathematics. Also, before

students begin the unit they will take a pre-test (Appendix C) to be later compared with a post-test (Appendix D). These results will present quantitative data as to whether students' problem-solving skills improved sufficiently. The numerical grade will reflect any change in the performance of the student. The pre- and post- tests are mirrors of each other, with slight change in number values, but no alterations in concepts or words.

Throughout the unit there will be many writing opportunities to observe and monitor the improvements of students' mathematical reasoning. Unit 3 will consist of a unit organizer (Appendix E). This organizer will guide the students through the chapter by encouraging them to analyze a problem in steps and justify their answer. After section four, students will be given a Prove Your Point (PYP) problem (Appendix F) to express their thought process and show their problem solving skills by defining the problem, analyzing what they already know and what need to know, solving the problem, and them reflecting on the "reasonableness" of their solution. Students will analyze their steps in solving the problem to justify their answer, asking themselves "Does my answer sound reasonable, does it make sense?" If students are able to rationalize a solution to himself/herself, then they are taking a step forward in their problem-solving skills.

After sections 2, 4, and 6, I will have the below-average students fill out a self-assessment (Appendix G). I will be given many chances to observe how the students are improving in their reasoning skills, but I also want to take note on how the students perceive their effort and improvement. I believe it is important to analyze the students' point of view since progress also lies in the confidence the student is gaining. Lastly, students will participate in a post-survey (Appendix B) to establish if there has been positive or negative change in the students' opinions of mathematics, writing in mathematics, and problem-solving skills.

Data Analysis

The action research data collected will be analyzed in a methodical and scientific manner. A pre-research survey will shed light on my student's initial attitude and viewpoint of writing in mathematics. The pre-test sets the knowledge and skill baseline that can later by compared to the post-test. The organizer is a tool that will allow me to follow the progress of each student's problem solving skills; at the end of each section, the organizer will be reviewed for skill development milestones. Analysis of the PYP will detail what methods students are using in problem solving and show how they justify their solutions. Self-assessments, at the end of every two sections, will indicate whether students are recognizing their improvement in problem solving skills. Finally, post-tests will be scrutinized, not only by the overall grade with regard to the pre-test, but by the particular learning objectives of the unit.

Each answer to questions within the pre-research survey and the self-assessment will be assigned a numerical value, allowing for a summation score and easy comparison. Pre and post-tests are scored traditionally, which will allow for obvious numerical comparisons to be made. Within the organizer and PYP, rubrics will be used to assess problem solving skill development, which will assist in drawing meaningful comparisons and trends. These data sets will be used to create graphical representations of compelling discoveries and evidential justification of improvement.

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Appendix A

Parent consent form: Students are under the age of 18 so I need parent/guardian consent

Dear Parent/Guardian,

I am conducting research on the impact of written communication in mathematics has on problem solving skills of students who are receiving a grade below C (average). The purpose of this study is to analyze how students' grades, attitudes, understanding, and problem skills improve by exploring the steps of a problem and justifying the answer. The results of the study may help teachers validate the use of writing as a form of communication in a mathematics classroom to enhance their students' ability to understand many concepts. It is determined by the first two chapter of this course that your child is receiving a letter grade below C. With your permission, I would like to ask your child to volunteer for this research.

A pre- and post-test of unit 3 will be given to gather quantitative date to show improvement. The unit organizer and a PYP worksheet will be analyze to determine an improvement in written communication. And a survey will be given every couple day to explore students' opinions of mathematics and their own work. Although I will ask the student to write their names on all papers exchanging between the student and me, their identity will be confidential. I will replace names with a number code and results will only be reported as group data.

You and your child have the right to withdraw consent for your child's participation at any time without consequence. There are no known risks or immediate benefits to the participants. If you have any questions about this research protocol, please contact me at (801)971-8368. Questions or concerns about your child's rights as research would like to obtain information or offer input, or would like to register a complaint about this study, you may contact the Michigan State University's Human Research Protection Program at (517) 355-2180, FAX (517) 432-4503, or email irb@msu.edu or regular mail at 207 Olds Hall, MSU, East Lansing, MI 48824.

I have read the information provided above and all of my questions have been answered. I voluntarily agree to the participation of my child in this study. I will receive a copy of this consent form for my information.

Sincerely,

Jennifer Heckman

_____ **_____**

Parent / Legal Guardian Signature **Parent / Legal Guardian Signature**

Name of Child _____

Appendix B

Unit 3: Pre/Post Student Survey: Survey I put together to what students think of mathematics and their problem solving skills

Please circle the word/words that best fits you for each statement.

1. I like math

Strongly Agree Agree Neutral Disagree Strongly Disagree

2. I am good at math

Strongly Agree Agree Neutral Disagree Strongly Disagree

3. It is important to have math skills for other areas other than math class

Strongly Agree Agree Neutral Disagree Strongly Disagree

4. I am able to show the work required to solve a math problem

Strongly Agree Agree Neutral Disagree Strongly Disagree

5. I am able to explain my answers with others

Strongly Agree Agree Neutral Disagree Strongly Disagree

6. I feel it is important to write out steps when solving a math problem

Strongly Agree Agree Neutral Disagree Strongly Disagree

7. I feel 100% positive with my solutions to math problems

Strongly Agree Agree Neutral Disagree Strongly Disagree

Appendix C

Pre-test: A test I put together to determine what students know before starting the unit

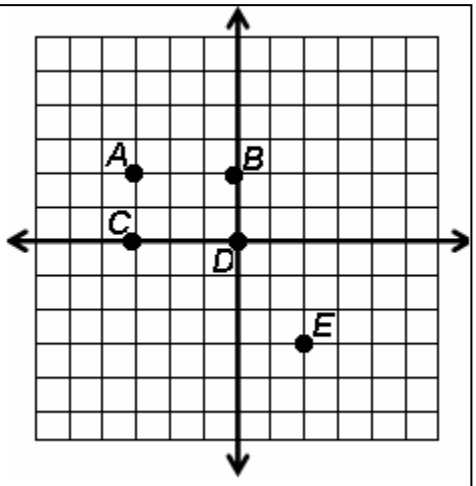
Integrated 1

Unit 3 Pre-Test

Multiple Choice Section 1 point each

***Directions**: Write the best answer choice on the answer sheet. There is no partial credit on this section.*

Use the graph to answer questions 1 – 4.

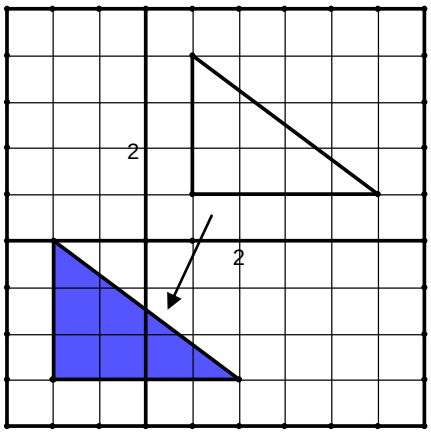


1. Which point on the graph is the origin?
2. Which point on the graph has the coordinates (2, -3)?
3. Which point on the graph is in Quadrant II?
4. Which point on the graph is on the horizontal axis?

5. The x-coordinate is negative in both Quadrants
   - **A.** I and II
   - **B.** II and III
   - **C.** III and IV
   - **D.** I and IV

6. The x- and y-coordinates are both positive in which Quadrant?
   - **A.** Quadrant I
   - **B.** Quadrant II
   - **C.** Quadrant III
   - **D.** Quadrant IV

7. The clear triangle is the original triangle. It has been translated and the translation is shown by the shaded triangle. Which best describes the translation?



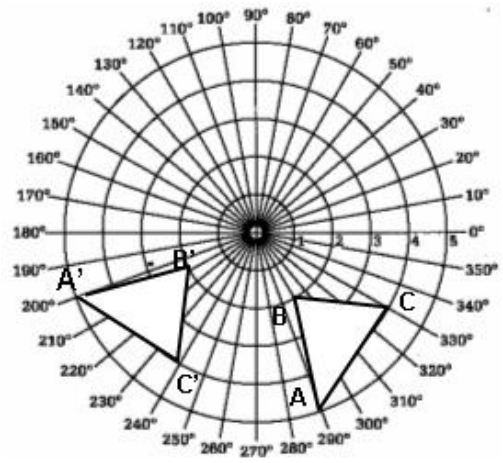
A. 3 units down and 4 units up

B. 3 units up and 4 units right

C. 3 units left and 4 units down

D. 3 units right and 4 units up

8. Which best describes the rotation of $\triangle ABC$ to $\triangle A'B'C'$?

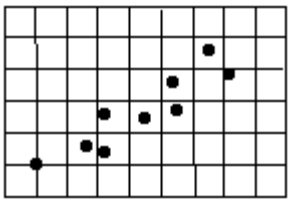


A. 200° counterclockwise

B. 200° clockwise

C. 90° counterclockwise

D. 90° clockwise



9. Which best describes the correlation of the points on the scatter plot?

A. positive correlation

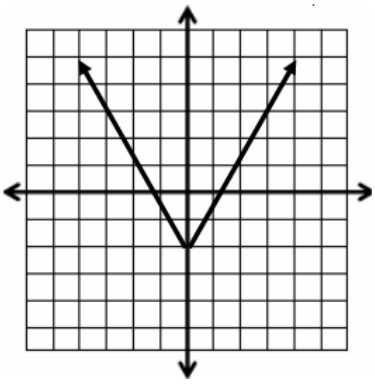
B. negative correlation

C. non-straight correlation

D. no correlation

Why did you choose your answer for #9?

10. Is the graph a function? Why or why not?



Use the paragraph below to answer questions 11 and 12.

A bag of chips costs $2.99. Your total grocery bill, *b*, is a function of the number of bags of chips, *n*, you purchase.

11. Write an equation to represent this function.

A. $b = \frac{2.99}{n}$

B. $n = 2.99b$

C. $\frac{b}{2.99} = n$

D. $b = 2.99n$

12. The dependent variable is

A. the number of bags of chips purchased

B. the cost of a bag of chips

C. your total grocery bill

D. the number of chips in each bag

13. Write an equation to represent the function.

A. $y = x + 1$

B. $y = x + 2$

C. $y = x - 2$

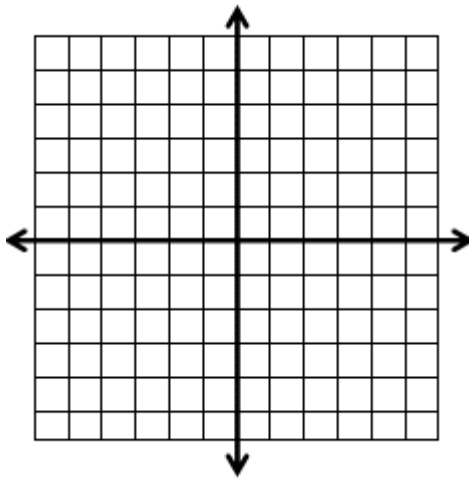
D. $y = x - 1$

x	*y*
-3	-1
-2	0
-1	1
0	2
1	3
2	4
3	5

Free Response Section ***4 points each***

Directions*: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as the accuracy of your results and explanation. Answers should be expressed in simplest form.*

14. a) Plot the points A(-1, 2), B(2,2), C(4,-1) and D(-3, -1). Connect the points in order, and connect from A to D and from D back to A.



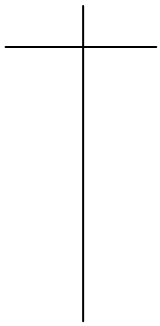
b) Name the figure formed.

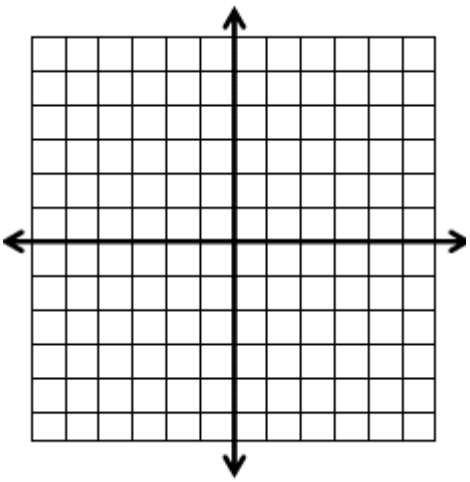
c) Find the area of the figure.

d) What process did you use? Why?

15. Make a table of 5 values & graph each function.

y = 3x



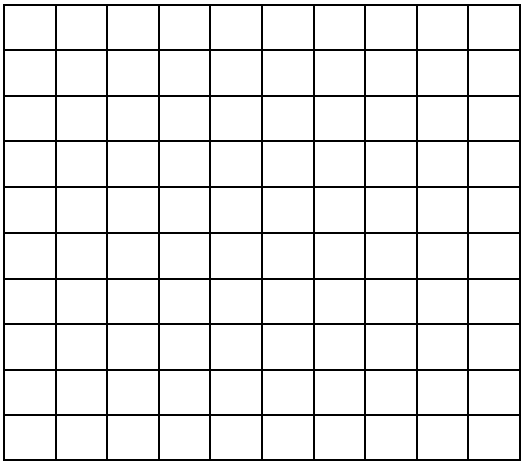


16. The table below contains homework scores and test scores from some of the students who took this class last year.

Remember to label your graph!!

a) Create a scatter plot.

Homework Percent	Test Percent
51	27
100	88
63	59
88	88
81	73
99	94
76	84
69	65



b) Draw a best fit line.

Appendix D

Unit 3 Post-test: A test I put together to compare scores with the pre-test. Scores will determine if students improved in their mathematical knowledge for unit 3

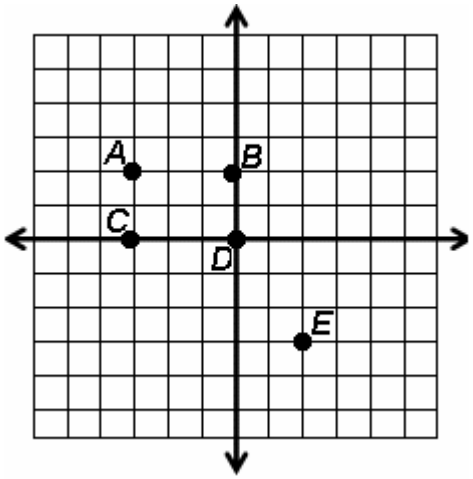
Integrated 1

Unit 3 Post-Test

Multiple Choice Section ***1 point each***

Directions*: Write the best answer choice on the answer sheet. There is no partial credit on this section.*

Use the graph to answer questions 1 – 4.



1. Which point on the graph has the coordinates (2, -3)?
2. Which point on the graph is the origin?
3. Which point on the graph is on the vertical axis?
4. Which point on the graph is in Quadrant II ?

5. The x- and y-coordinates are both positive in which Quadrant?
   - **A.** Quadrant I
   - **B.** Quadrant II
   - **C.** Quadrant III
   - **D.** Quadrant IV

6. The x-coordinate is positive in both Quadrants
   - **A.** I and II
   - **B.** II and III
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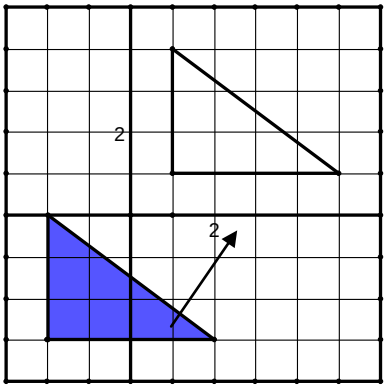
7. The shaded triangle is the original triangle. It has been translated and the translation is shown by the clear triangle. Which best describes the translation?



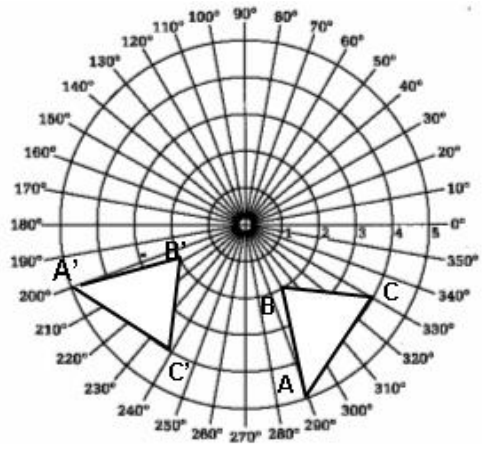
A. 3 units left and 4 units down

B. 3 units right and 4 units up

C. 3 units down and 4 units up

D. 3 units up and 4 units right

8. Which best describes the rotation of ΔABC to $\Delta A'B'C'$?



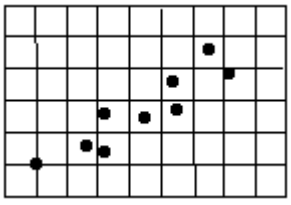
A. 90° clockwise

B. 90° counterclockwise

C. 200° clockwise

D. 200° counterclockwise

9. Which best describes the correlation of the points on the scatter plot?



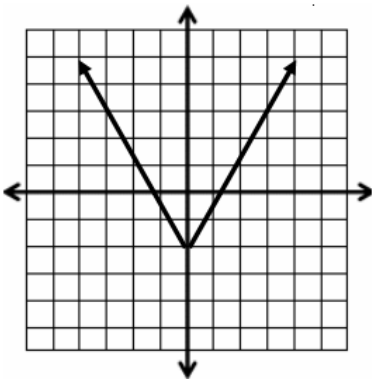
A. negative correlation

B. positive correlation

C. non-straight correlation

D. no correlation

10. Is the graph a function? Why?



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A. $\frac{b}{2.99} = n$

B. $b = 2.99n$

C. $b = \frac{2.99}{n}$

D. $n = 2.99b$

12. The control variable is

A. the number of bags of chips purchased

B. the cost of a bag of chips

C. your total grocery bill

D. the number of chips in each bag

13. Write an equation to represent the function.

A. $y = x + 1$

B. $y = x + 2$

C. $y = x - 2$

D. $y = x - 1$

x	*y*
-3	-4
-2	-3
-1	-2
0	-1
1	0
2	1
3	2

Free Response Section ***4 points each***

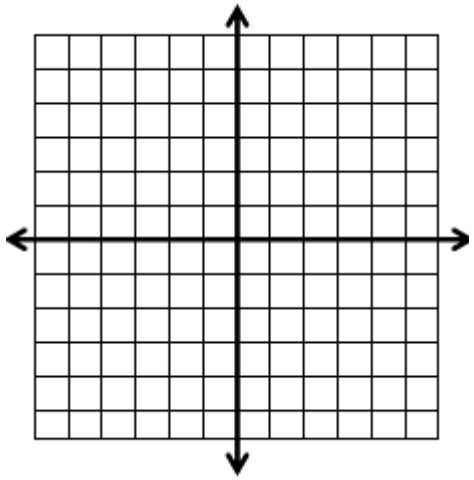
Directions*: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as the accuracy of your results and explanation. Answers should be expressed in simplest form.*

14. a) Plot the points A(-1, 2), B(2,2), C(4,-2) and D(-3, -2). Connect the points in order, and connect from A to D and from D back to A.

b) Name the figure formed.

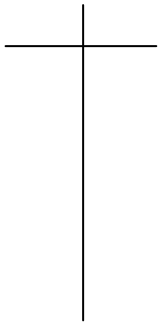
c) Find the area of the figure.

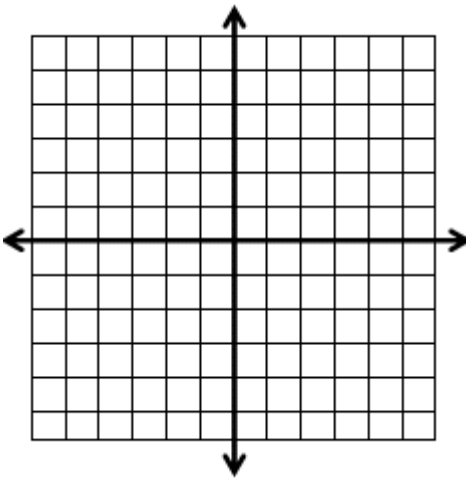
d) What process did you use? Why?



15. Make a table of 5 values & graph each function.

y = -3x



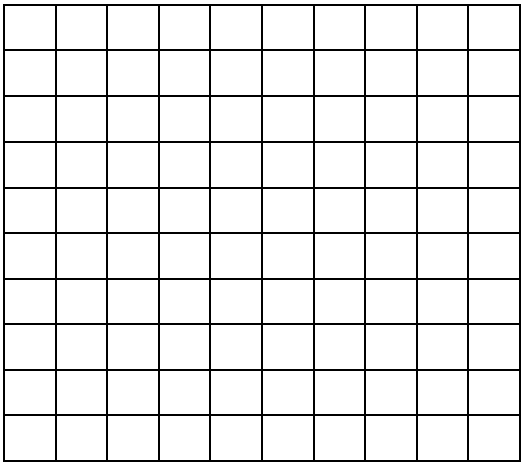


16. The table below contains homework scores and test scores from some of the students who took this class last year.

Remember to label your graph!!

a) Create a scatter plot.

Homework Percent	Test Percent
84	75
99	92
96	80
62	60
81	73
99	94
76	84
69	65



b) Draw a best-fit line.

Appendix E

Organizer: This organizer was put together by the high school algebra teachers of 2011 at Mountain View High School, Vancouver, Washington. The organizer will be used to guide the students through unit 3. I will use the organizer to analyze how students are problem solving

Name:

Date:

Unit 3:

Coordinates and Functions

Mountain View High School
Vancouver, Washington
2010

Rubinstein et. al. (2002). *Integrated mathematics: 1.* Illinois: McDougal Littell.

Integrated 1

Unit 3 Benchmarks

GLE	Text Page	Organizer Page	Topic
1.3.3			Graphing in a plane: Quadrants
			Vertical and horizontal axes
			Origin
			Coordinates
1.3.3			Graph polygon and find its area
			Definition of polygons Triangle
			Trapezoid
1.3.4			TRANSFORMATIONS Translations Translate a triangle – up/down and/or left/right
			Rotations Center of rotation
			Rotate a polygon – in quadrants and on polar graph paper
1.4.5			Scatterplots Draw a fitted line
			Positive, negative or no correlation
1.4.6			Functions Dependent and Control Variables
			Vertical line test
1.4.3			Graphing equations with points
			Square function, absolute value, identity & reciprocal functions

3-1: Coordinates for Locations

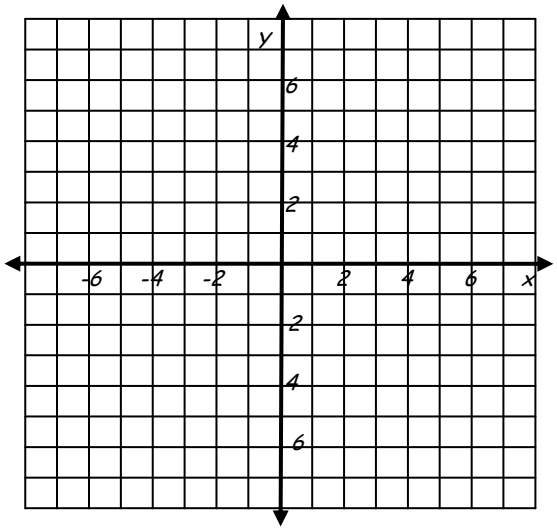
Summarize the focus of this section.

Skim the section.

What are two things in this section that are familiar to you?	What are two things in this section that are new to you?

Coordinate Systems

Read about Coordinate Systems on **page 184** in your textbook.



On the coordinate plane to the left, label the following items as they are labeled in the textbook.

1. the 4 quadrants
2. the vertical axis
3. the horizontal axis
4. the origin
5. the point (2, -4)
6. the point (3, 1)

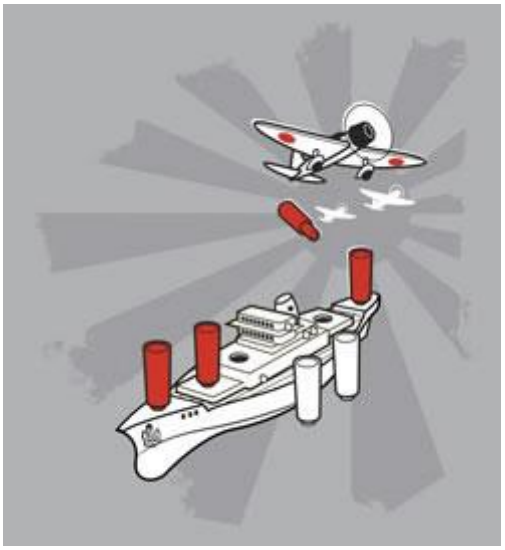
	Talk-It-Over, page 184
5	
6	

7	
8	
9	

Read about the map coordinates at the bottom of page 184. Answer the questions to the Sample on page 185.

a.	
b.	

BATTLESHIP



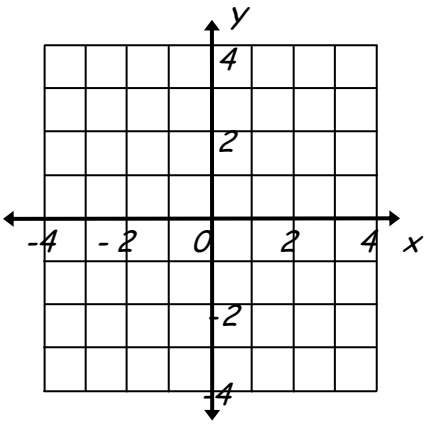
On this page, you will play the game Battleship. Your mission – to sink your opponent's ships by "hitting" them.

On the following grid on the left, place the following ships by drawing them along a vertical or horizontal line. Your teacher will give you further instructions about this.

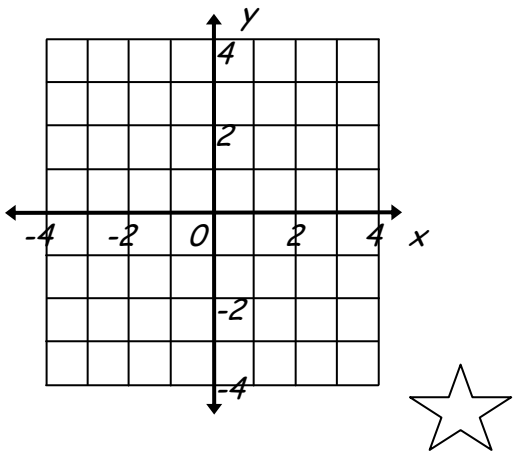
Once you and your opponent have both placed all of their ships on your graphs, you will take turns giving ordered pairs in an attempt to hit their ships. You will find it helpful to keep track of the ordered pairs you call out on the second graph provided. If you state an ordered pair that your opponent has covered with a ship, then you have hit that ship. Your opponent will tell you if you have hit a ship or missed all of them. After you have hit all of the ordered pairs covered by that ship, your opponent will tell you that the ship is sunk. Once one player has sunk all of the opponent's ships, the game is over.

Ships: Aircraft Carrier: 5 points
Battleship: 4 points
Destroyer: 3 points
Submarine: 3 points
Patrol Boat: 2 points

YOUR GRAPH



YOUR OPPONENT'S GRAPH



3-2: Introduction to Coordinate Geometry

Summarize the focus of this section.

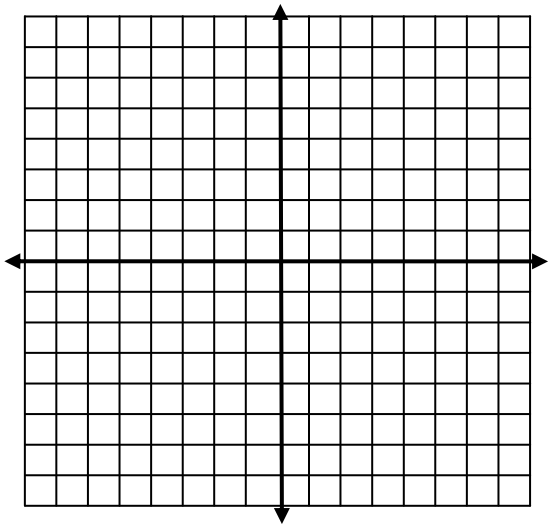
Skim the section.

What are two things in this section that are familiar to you?	What are two things in this section that are new to you?

When you draw a shape on a coordinate plane, analyze it, find its area, or compare it to another shape, you use coordinate geometry.

Sample 1

a. Plot the points:
A(5, 1), B(2, 1), C(2, -2)
Connect the points in order and connect the last point to the first.



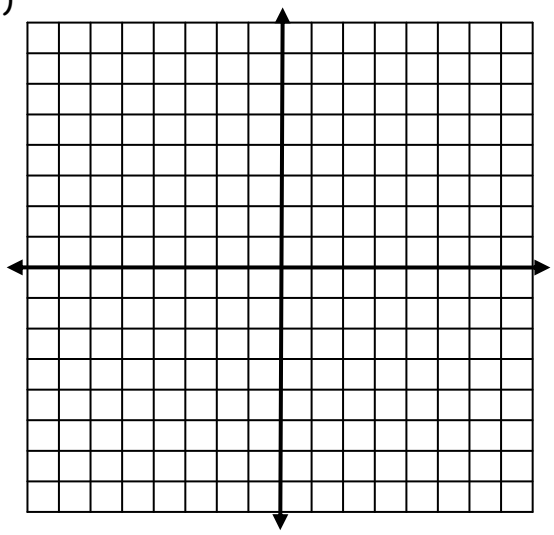
What is the specific name of the polygon formed?_____

Explain how you know what it is. _____

_____

_____

b. Plot the points:
D(3, 1), E(1, 3), F(-2, 3), G(-3, -1), H(-1, -3), J(2, -3)



Connect the points in order and connect the last point to the first.

What is the specific name of the polygon formed?_____

Explain how you know what it is. _____

_____

_____

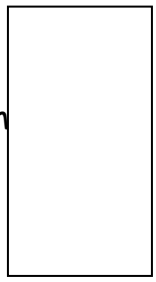
When you're looking for **area** of a shape, what does that mean? _____

_____

Find the area of the following rectangles.

1. 3 cm

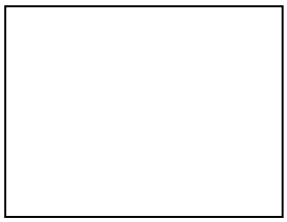
7 cm



Area = _____

2. 6 cm

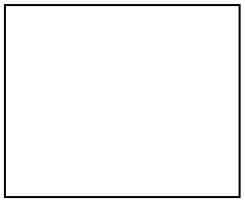
5 cm



Area = _____

3. b

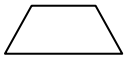
h



Area = _____

What is the formula for the area of a rectangle? Area = _____

A triangle is defined to be a three sided polygon. Example: 

A trapezoid is defined as a quadrilateral with exactly one pair of parallel sides. Example: 

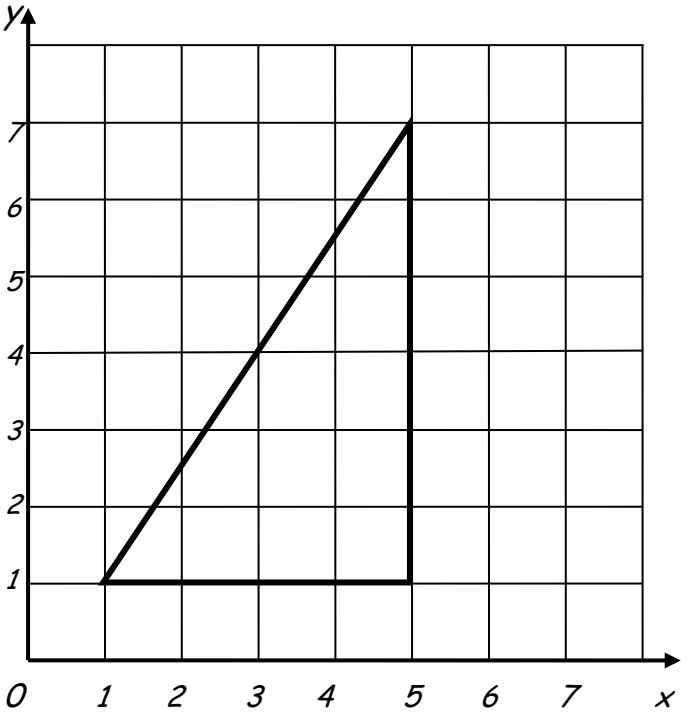
Use that fact to find the area of a right triangle and the area of a right trapezoid (a trapezoid with a right angle in it).

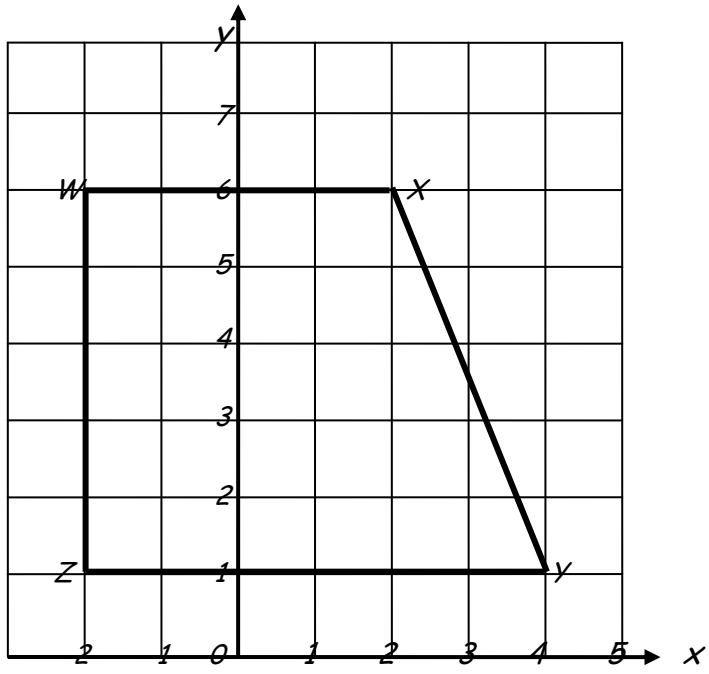
On the next page, you will find a right triangle and a right trapezoid. Using cutting, taping, duplicating, gluing, folding or any other method you choose, you and a partner will find the area of the shapes and create a poster to show how you figured it out.

Note: You may need to look up some vocabulary in your textbook to be successful on tonight's homework. You will find the names of shapes in your textbook on page: _____

FINDING THE AREA OF TWO SHAPES

Find the area of the following two shapes and create a poster to show how you figured out the area of each of them. Ask your teacher if there are extra copies available.





3-3: Translations

Summarize the focus of this section.

Skim the section.

What are two things in this section that are familiar to you?	What are two things in this section that are new to you?

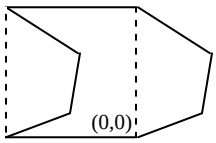
EXPLORATION:

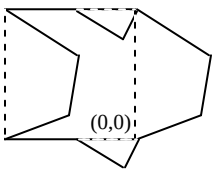
Materials:
- A grid provided by your teacher
- Scissors
- Tape
- A large coordinate plane on page 87.

In the following activity, you will need to make your grid into a design with 8 to 12 corners, as follows. Label the bottom right-hand corner of the shape (0,0), as shown in the example below. Now, cut a shape out of one edge, shift it to the other side of the grid, and tape it back on. Then, do the same out of one edge you have not yet altered.

Example:

original: 

after 1st alteration: 

after 2nd alteration: 

Put (0, 0) on the corresponding point on the grid below, and trace your shape. Shift your shape so it fits in another quadrant, and trace it again. Do that 2 more times until your shape fits together without gaps, and is located in all 4 quadrants.

You have now created a **tessellation** -- a space that is filled by a single shape with no gaps.

Explain how you moved your piece from the original position in Quadrant II to its new position in

a. Quadrant I: Direction: _____

Distance: _____

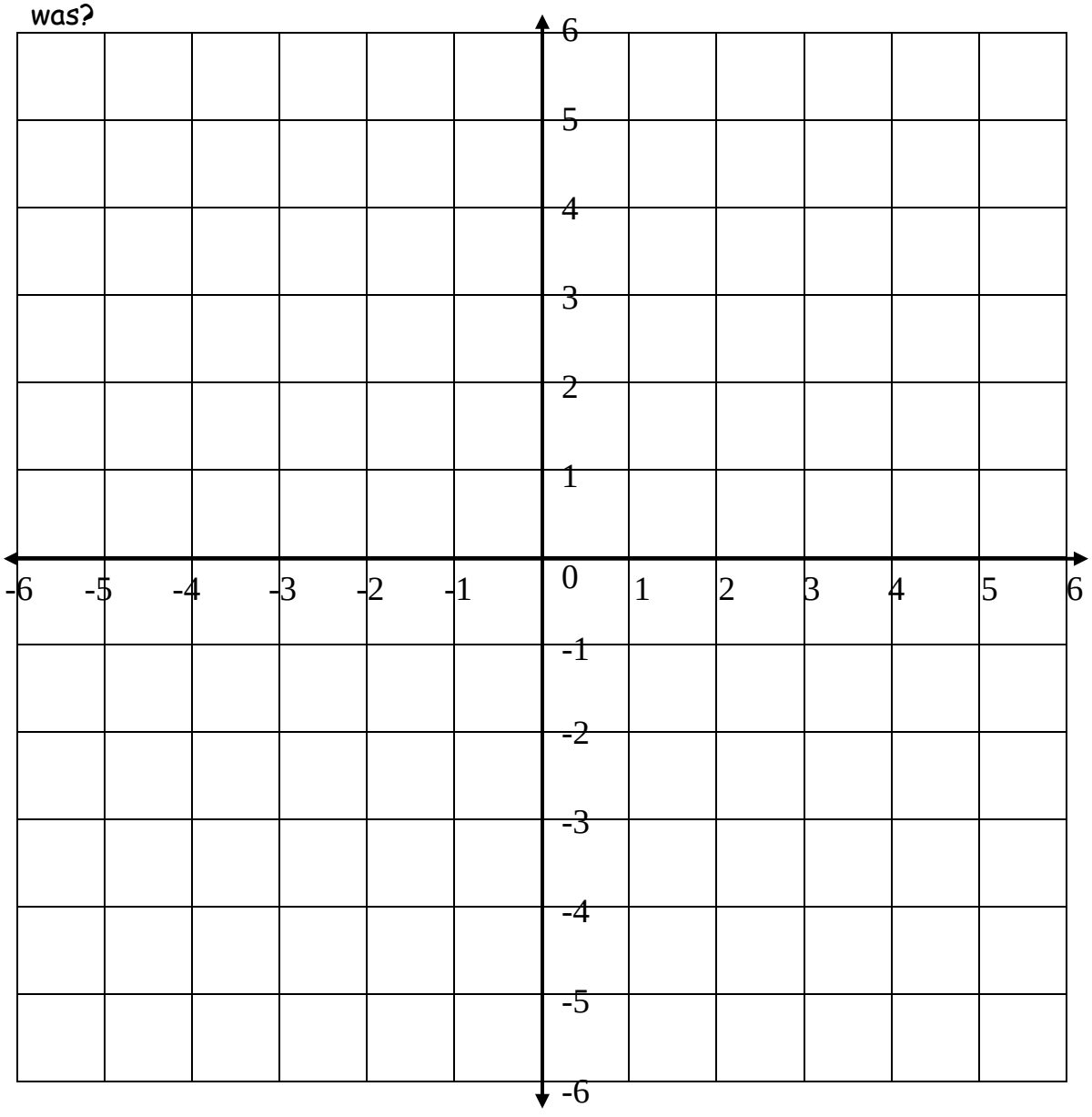
b. Quadrant III: Direction: _____

Distance: _____

c. Quadrant IV: Direction: _____

Distance: _____

Could you have predicted from your original square what the distance for each move was?



On page 197, you will find the definition for translation. Please write it below.

translation: _____

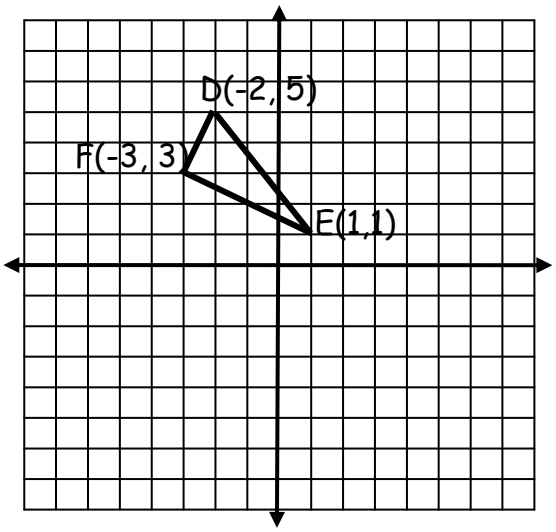
_____

	Talk-It-Over, page 197
1	
2	
3	

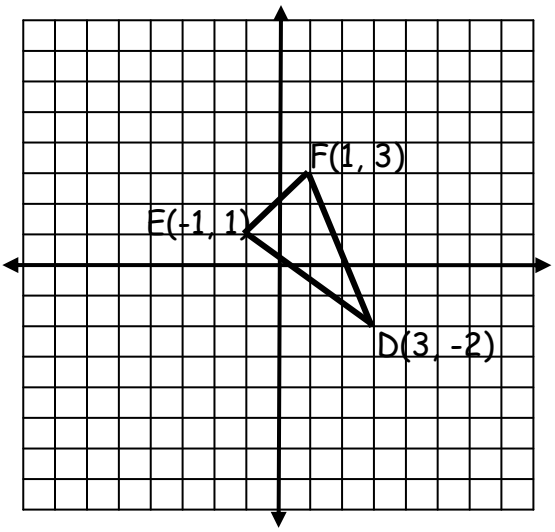
Read Sample 1 on page 198. Try it and the Sample 1b below.

Find the coordinates of each vertex of ΔDEF after the translation:

a. 6 units right and 2 units down



b. 4 units left and 5 units up



	Talk-It-Over, pg. 198
4	
5	

Translational Symmetry

Name three places where you would find examples of translational symmetry:

1. _____

2. _____

3. _____

How do the coordinates of the vertices of a polygon change for each of these translations?

a. a number of units right or left: _____

b. a number of units up or down: _____

c. a number of units right or left *and* a number of units up or down: _____

_____

3-4: Rotations

Summarize the focus of this section.

 Skim the section.

What are two things in this section that are familiar to you?	What are two questions you have about the information in this section?

Read the bottom of **page 202** and define the following words:

rotation: _____

center of rotation: _____

The center of rotation can be located _____

_____ or _____.

transformation: _____

When you rotate an object, does its size or shape change? _____

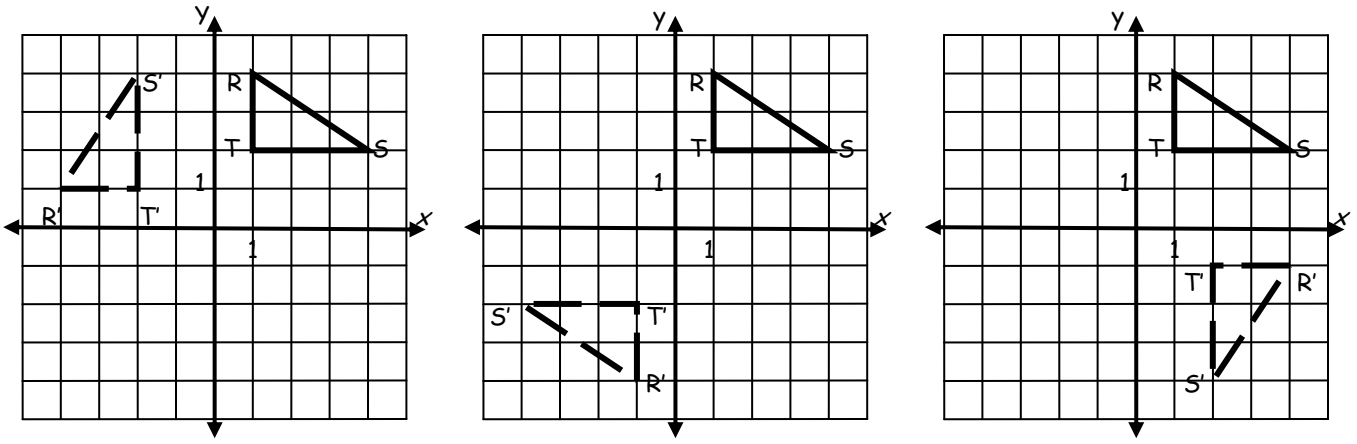
In order for someone to copy a rotation, they need to know which direction to turn the object, how far to rotate the object, and where the center of rotation is located. The direction can be *clockwise* (↻) or *counterclockwise* (↺). How far to rotate the object can be given in degrees or as a fraction.

Sample 1B: Read and study Sample 1 from the textbook, and use that knowledge to help you do the following problems.

The graphs show rotations of ΔRST around the origin. Describe the direction and amount of the rotation of each graph.

_____° _____ _____° _____ _____° _____

_____° _____ _____° _____ _____° _____



	Talk-It-Over, page 204
3	

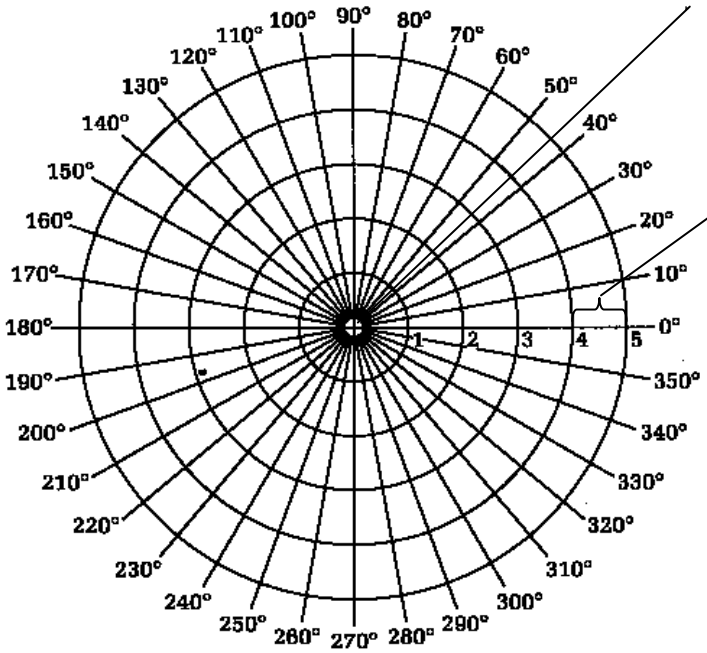
4a	
4b	

Copy the information from the yellow box on **page 204** about rotations:

To describe a rotation, you need to tell:

- _____
- _____
- _____

Drawing rotations is easier on *polar graph paper*. Fill in the blanks about information you need to know about polar graph paper.



_____

_____

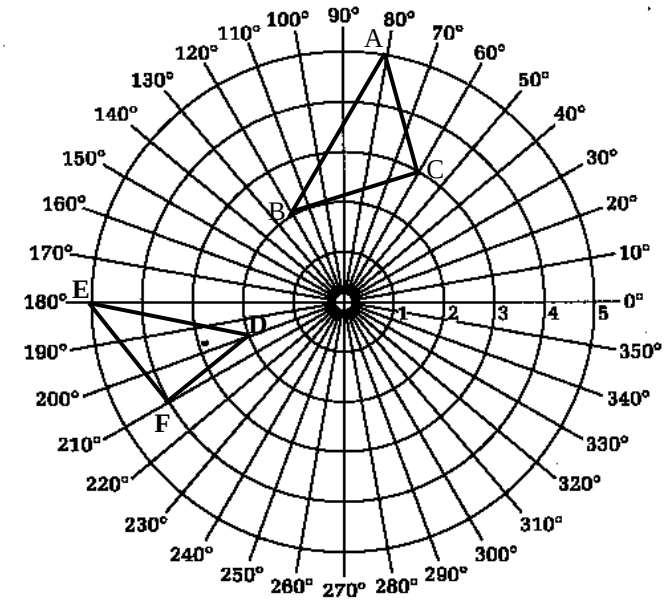
_____

_____

_____

_____

_____



Study Sample 2 on page 205, and rotate ΔDEF 60° clockwise around the origin.

Now, rotate ΔABC 50° counterclockwise around the origin.

Rotational Symmetry

Read the paragraph about rotational symmetry on page 205.
Explain what **rotational symmetry** is in your own words: _____

_____

Sample 3:

Determine whether or not each figure has rotational symmetry.

a. _____ b. _____ c. _____

How are transformational and rotational symmetry alike? _____

_____

How are they different? _____

3-5: Scatter Plots

Summarize the focus of this section.

EXPLORATION

Materials: Overhead graph paper and overhead markers, measuring tapes

1. Predict what the relationship between height and lower-arm length might be:

_____

2. Fill in the chart for the people in your group.

Name	Height (cm)	Lower-Arm Length (cm)

3. Plot your data on the transparency provided by your teacher. Put your graph on the overhead, stacked with the others.

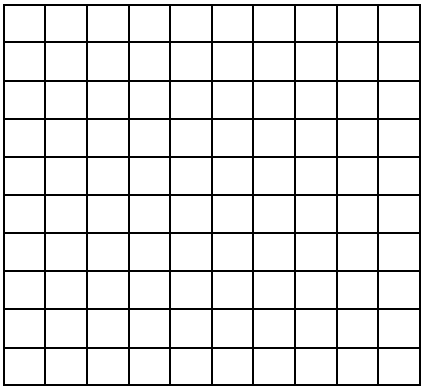
4. Describe any trends you see in your data. Does your graph support the prediction you made in step 1? _____

_____

5. Display your table and graph along with those of other groups. How are the data and the graphs alike or different? _____

_____

6. Plot the data from all the groups on this graph.



7. Does the graph support the prediction you made in step 1? _____

_____

8. Suppose a student is 171 cm tall. Use the graph to predict the student's lower arm length. _____

Drawing a Fitted Line

Read the bottom half of page 212 about drawing a fitted line.

Define a **fitted line**: _____

How do you know from your fitted line if there is a strong or weak correlation between two data sets? _____

_____

Would the following examples have a strong, weak or no correlation?

a. The number of assignments given and the number of problems on the assignment.

   _____

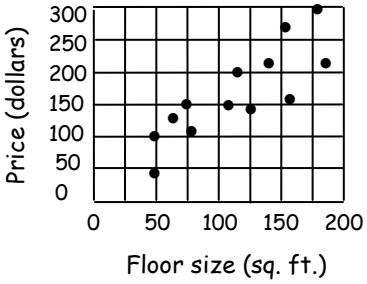
b. Homework grade and test grade _____

c. Number of cavities and age _____

d. Price of gas and distance you'd drive on vacation _____

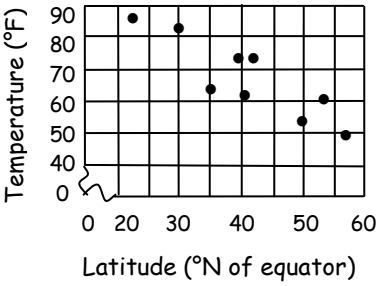
The following are examples of *scatter plots*, and they are used to show the relationship between two data sets.

Tent prices



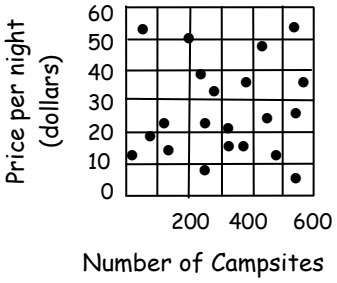
When two data sets increase together, they have a **positive correlation.**

October High Temps



When two data sets decrease together, they have a **negative correlation.**

Campground Prices



Sometimes data sets show **no correlation.**

	Talk-It-Over, page 212
1	
2	

The table shows the piston displacement and horsepower (hp) of nine similarly powered cars. What horsepower do you predict for a 3200 cc engine?

Car	A	B	C	D	E	F	G	H	I
Displacement(cc)	2389	1836	3405	3535	1840	2977	2986	1998	2164
Horsepower(hp)	155	134	300	310	140	270	220	119	130

Make a scatter plot of the data below. Be sure to title the graph, label and scale the axes, draw a fitted line, and make a prediction.

Answer the questions to the Talk-It-Over on page 213.

	Talk-It-Over
3	
4	
5a.	
5b.	

3-6: Graphs and Functions

Summarize the focus of this section.

 Skim the section.

What are two things in this section that are familiar to you?

What are two things in this section that are new to you?

	Talk-It-Over, page 218
1	
2	
3	
4	
5	

Dependent and Control Variables

Read the paragraph at the bottom of page 218.

Define **dependent variable**: _____

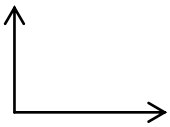
Define **control variable**: _____

Read Sample 1 and study its response.

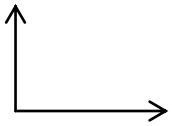
	Talk-It-Over, page 219
6	
7	

In problems 1-5, identify the dependent variable and the control variable in each situation. Then put a title on the graph and label the axes.

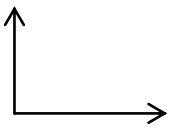
1. Jerry is driving his car at 45 mi/h. The longer he drives, the farther away from home he is. Control: _____
   Dependent: _____



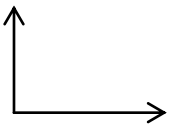
2. Kim is reading a biography of Marie Curie. The more chapters she reads, the fewer pages she has to go to finish the book.
   Control: _____
   Dependent: _____



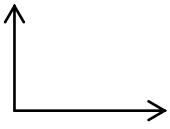
3. An object falls at a speed of 125 ft/sec. An object that falls farther falls longer.
   Control: _____
   Dependent: _____



4. Mrs. Seamstress likes to make quilts. She needs more fabric to make bigger quilts.
   Control: _____
   Dependent: _____



5. A football player throws 50 passes during practice. The more passes he throws, the slower they go.
   Control: _____
   Dependent: _____



Functions

Read the top two paragraphs on page 220.

Define function: _____

_____

List at least 4 ways that functions can be represented:

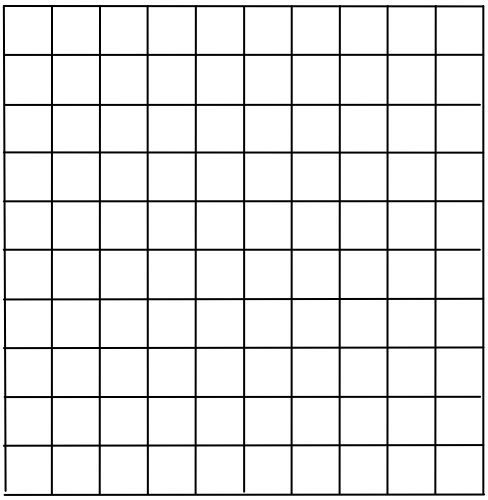
a. _____ b. _____

c. _____ d. _____

Read Sample 2 on page 220. Then try the following problem.

Tracy is going to save some of the earnings from her summer job. She plans to save $1 the first week, $2 the second week, and for the next 5 weeks, match her savings to her total savings from the preceding weeks. Complete the chart and draw a graph to show her weekly savings for the seven week period. Tell whether the graph represents a function.

Week	Savings ($)
1	1
2	2
3	3
4	6 (why?)
5	
6	
7	

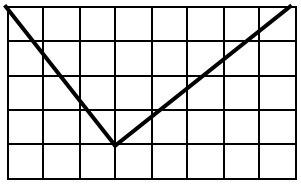


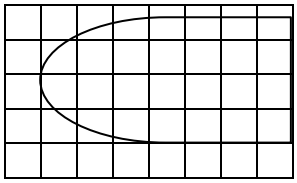
Is this a function? _____

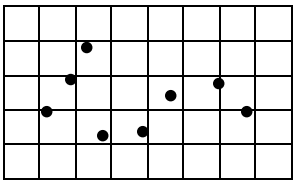
Vertical Line Test for Functions

There is an easy way to look at a graph and determine whether it represents a function or not. Read the bottom of page 220 and the top of page 221 about the vertical line test.

Read through Sample 3 and use the vertical line test to determine whether the following are functions.







3-7: Functions and Equations

Summarize the focus of this section.

 Skim the section.

What are two questions you have about topics in this section?

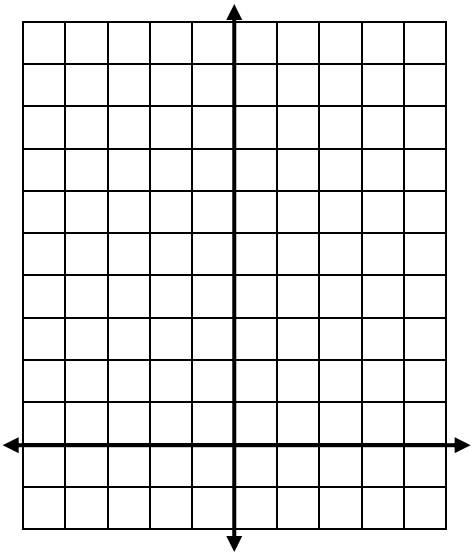
EXPLORATION: Is the relationship between a number and its square a function?

Materials: graph paper, graphics calculator

1. Complete the table.

Number	Square
x	x^2
-3	9
-2.5	6.25
-2	4
-1.7	
-1.3	
-1	
-0.5	
0	
.5	
1	
1.3	
1.7	
2	
2.5	
3	

2. Plot the ordered pairs on a coordinate plane.



3. Describe the shape the points seem to make. _____

   _____

4. Does the relationship between x and x^2 appear to be a function? _____

   Why or why not? _____

5. Choose four values between -3 and 3 that are not already on the table. Find their squares and plot the points. _____

6. You cannot square all the numbers between 3 and -3 and plot the ordered pairs. However, your graphics calculator can do many more than you can in a much shorter time. Enter the equation $y = x^2$ on the "y = " screen of the calculator. Compare the result with your graph.

Writing Equations

Read about writing equations at the top of page 226.

The _____. the _____, the _____ and the _____ all represent the same relationship.

Read through Sample 1 and its response on page 226.

See if you can figure out the equation to represent the function given in the table.

x	y
-5	-10
-3	-6
-2	-4
-1	-2
0	0
1	2
4	8
6	12

Function: _____

Read through Sample 2 and its response on pages 226-227.

Write an equation to represent the function. Be sure to tell what each variable represents.

To mail a letter to Mexico, Felipe pays $0.35 cents for the first half ounce and $0.10 for each additional half ounce.

Equation: _____

	Talk-It-Over, page 227
1	
2	

Graphing Equations

Read about graphing equations on page 227 of your textbook.

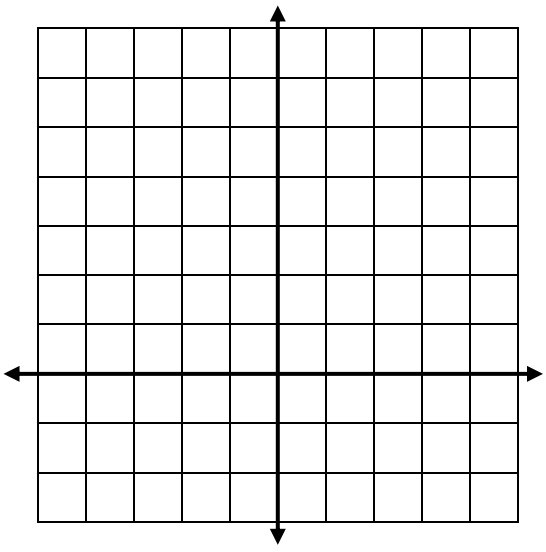
You can graph a function represented by an equation by using _____,

_____, or _____.

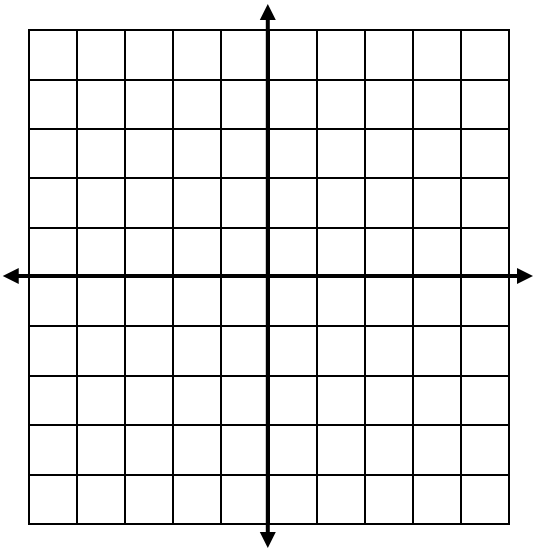
To graph using graph paper, make a table of values and plot the ordered pairs. If all of the values between the ordered pairs make the equation true, then the points can be connected.

Graph each of the following equations.

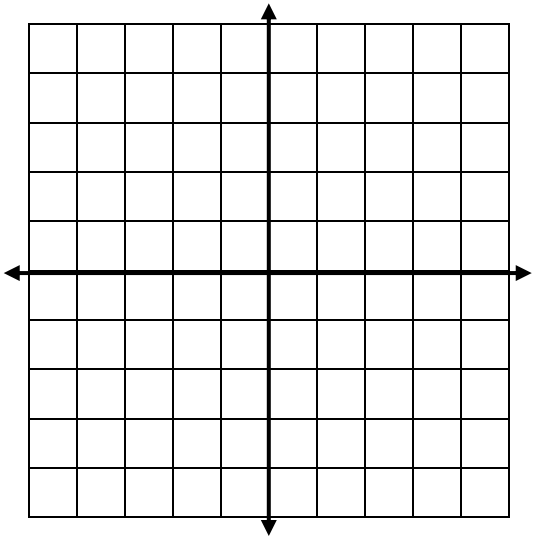
y = |x|		
x	y	(x, y)
-3	|-3| = 3	(-3, 3)
-2	|-2| =	(-2,)
-1	|-1|	(-1,)
0	| |	(,)
1	|1| = 1	(1 ,)
2	| |	(,)
3	| |	(,)



y = x		
x	y	(x, y)
-2		
-1		
0		
1		
2		

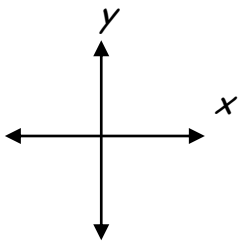
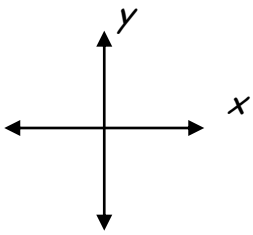
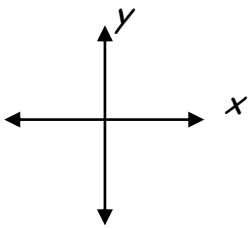
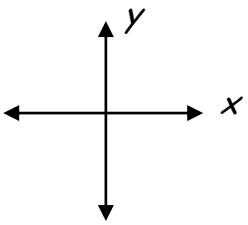


$y = \frac{1}{x}$		
x	y	(x, y)
-3		
-2		
-1		
-0.25		
-0.5		
0		
0.25		
0.5		
1		
2		
3		



Summary:

Sketch what each of the graphs you drew look like on the graphs below:

Equation	$y = x^2$	$y = \|x\|$	$y = x$	$y = \frac{1}{x}$
Words	Squaring function	Absolute value function	Identity function	Reciprocal function
Graph				

Suppose you are given a function described by an equation. How can you represent it

with a table? _____

with a graph? _____

✍ Unit 4 Notes

Appendix F

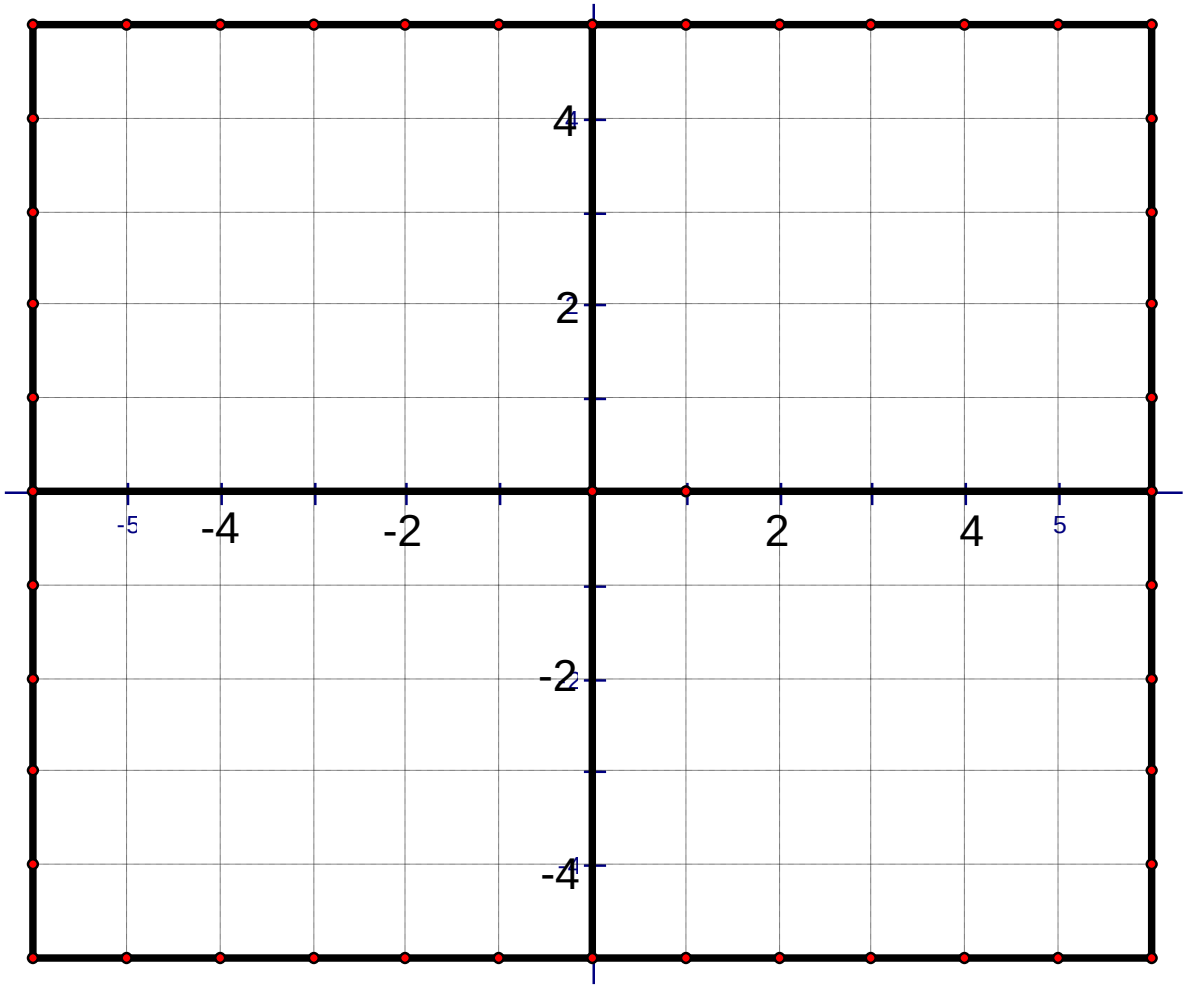
PYP: I put together this Prove Your Point to analyze how students break down a problem and justify his/her answer

Processing Your Process

Name:

Graph the points and find the area of the figure formed when the points are connected in order.

(2, 5), (-4, 5), (-4, -3), (0, 1), (2, -2), (2, 5)



What is the task?

What do I know about?

Solve Problem

Reasonableness and Correctness

Appendix G

Student Self-Assessment: I made this assessment for students to communicate with me what they think of their progress

Student Self-Assessment

Name:

Please circle the answer that best fits you.

1. I have written done my thought processes when solving math problems in Unit 3

Strongly Agree Agree Neutral Disagree Strongly Disagree

2. I have improved my process in finding solutions in Unit 3

Strongly Agree Agree Neutral Disagree Strongly Disagree

3. I feel confident on how I solve problems in Unit 3

Strongly Agree Agree Neutral Disagree Strongly Disagree

4. I have correctly answered problems in Unit 3

Strongly Agree Agree Neutral Disagree Strongly Disagree

5. I have developed skills that will help me solve problems in the future

Strongly Agree Agree Neutral Disagree Strongly Disagree

6. Writing steps to find a solution has been helpful

Strongly Agree Agree Neutral Disagree Strongly Disagree

7. Justifying a solution as made me more confident in my mathematics skills

Strongly Agree Agree Neutral Disagree Strongly Disagree